

SMT MMK COLLEGE OF COMMERCE AND ECONOMICS

SYJC PRELIMINARY EXAMINATION, JANUARY 2019

DATE:- 14/01/2019

SUB:- MATHS AND STATS

MARKS: - 80

TIME:- 3 HOURS

SECTION - I

Q-1) Attempt any SIX of the following :

[12]

i) If $A = \begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}$, $A^2 - 3A$

ii) If $y = \tan^{-1}(3x)$, find $\frac{d^2y}{dx^2}$

iii) Write down the truth values of the following statement:

a) 2 is a prime number b) Two parallel lines meet at a point

iv) If $x = \sin^2 \theta$, $y = \tan \theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ v) Evaluate $\int \frac{\cos(x+a)}{\sin x} dx$ vi) Evaluate $\int x^2 e^x dx$

vii) Find the area of the region bounded by the curve $y = \sqrt{6x+4}$, the x-axis, the lines $x=0$ and $x=2$

viii) Find the values of x for which the function $f(x) = x^3 - 6x^2 - 15x + 1$ is increasing.

Q-2. (A) . Attempt any TWO of the following :

[6]

i) Examine whether the following statement is a tautology, contradictions or neither.

$$(\sim p \vee q) \rightarrow [p \wedge (q \vee \sim q)]$$

ii) If $A = \begin{bmatrix} 5 & 2 \\ -3 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -3 & 4 \\ 5 & 8 & -2 \end{bmatrix}$ then prove that $AB \neq BA$. Find $A^{-1} B^{-1}$, if possible.

iii) $x^y = e^{x^y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

Q-2.(B) Attempt any TWO of the following :

[8]

i) Evaluate $\int \frac{7x+3}{\sqrt{3+2x-x^2}} dx$

ii) If 'f' is continuous on $(-\pi, \pi)$, where

$$f(x) = -2 \sin x, \quad \text{for } -\pi < x \leq -\frac{\pi}{2}$$

$$= \alpha \sin x + \beta \quad \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$= \cos x \quad \text{for } \frac{\pi}{2} \leq x \leq \pi. \quad \text{Find } \alpha \text{ and } \beta.$$

iii) A manufacturer can sell x items at a price of Rs $(330-x)$ each. The cost of producing x items is $C = x^2 + 10x + 12$. Determine the number of items to be sold so that the manufacturer can make maximum profit.

Q-3. (A) . Attempt any TWO of the following : [6]

i) Express following equations in matrix form and solve them by method of reduction.

$$x + 3y + 3z = 12, \quad x + 4y + 4z = 15 \quad \text{and} \quad x + 3y + 4z = 13.$$

ii) If $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$ show that $\frac{dy}{dx} = \cot(\theta/2)$

iii) If p and q are true and r and s are false statements, find the truth values of the following statements :

$$(a) (p \wedge q) \vee r \quad (b) p \wedge (r \rightarrow s) \quad (c) (p \vee s) \leftrightarrow (q \wedge r)$$

Q-3.(B) Attempt any TWO of the following : [8]

i) Evaluate $\int_2^3 \frac{\sqrt{5-x}}{\sqrt{x} + \sqrt{5-x}} dx$

ii) Find $\frac{dy}{dx}$ if $y = (\cos x)^{\log x} + (\log x)^x$

iii) Show that $f(x) = x^x$ is minimum when $x = \frac{1}{e}$.

SECTION -II

Q-4) Attempt any SIX of the following : [12]

i) A salesman is appointed on a fixed monthly salary of Rs 1500 together with a commission at 5% on the sales over Rs 10,000 during a month. If his monthly income is Rs 2050, find his sales during that month.

ii) For an immediate annuity paid for 3 years with interest compounded at 10% p. a., the present value is RS 10,000. What is its accumulated value after 2 years.

iii) A car worth Rs 3,80,000 is insured for Rs 2,50,000. In an accident it is damaged to the extent of Rs 76,000. Find the amount of compensation that can be claimed under the policy.

iv) For a bivariate data $\sum(x - \bar{x})(y - \bar{y}) = 121$ and $n = 10$. Calculate covariance between x and y .

v) Following are the two regression equations for a bivariate data, $8x - 10y + 66 = 0$ and $40x - 18y - 214 = 0$. Find the mean values of x and y .

vi) Three balanced coins are tossed simultaneously. If X denotes the number of heads, find the probability distribution of X .

vii) If X has poisson distribution with parameter $m=1$; Find $P(X \leq 1)$. [use $e^{-1} = 0.368$]

viii) Give the p.d.f. of a continuous random variable X as

$$f(x) = \frac{x^2}{3}, \quad -1 < x < 2$$

$$= 0, \quad \text{otherwise.}$$

Determine the c.d.f. of X and hence find $P(X < 1)$.

Q-5. (A) . Attempt any TWO of the following :

[6]

i) Calculate CDR from the data given below :

Age group	No. of persons (in 000)	No. of deaths
Below 10	12	150
10- 30	20	10
30-45	35	380
45-70	24	210
Above 70	15	540

ii) The ratio of prices of two cycles was 16:23. Two years later when the price of the first cycle has increased by 10% and that of second by Rs 477, the ratio of the prices become 11:20, find the original prices of two cycles.

iii) Solve the following minimal assignment problem

	A	B	C	D
I	3	4	6	5
II	5	6	10	9
III	1	2	3	2
IV	4	10	6	4

Q-5 (B) . Attempt any TWO of the following :

[8]

i) Complete the life table for the following data:

X	0	1	2	3	4	5
l_x	30	26	18	10	4	0

ii) If for a bivariate data, $u = \frac{x-70}{10}$ and $v = \frac{y-60}{20}$ and $\sum u = 60$, $\sum v = 40$, $\sum u^2 = 4260$, $\sum v^2 = 1720$.

$\sum uv = 1150$ and $n=10$, find the regression coefficient of x on y and y on x. Also find the value of 'r'.

iii) Two products A and B are available at the cost of Rs 30 and Rs 20 per pack respectively. Food A contains 80 and 9 units of proteins and vitamins respectively and Food B contains 40 and 5 units of proteins and vitamins respectively. Find how many packs of A and B must be purchased so as to meet the requirement of 600 units of proteins and 72 units of vitamins at the minimum cost.

Page.3

Q-6. (A) . Attempt any TWO of the following :

[6]

- i) In a certain city 20% of persons are vegetarians. If 5 persons from the city are chosen at random, find the probability that (a) none is vegetarian (b) at least one is vegetarian.
- ii) On an average a company produce three defective bikes everyday . Find the probability that the company produces (a) no defective bikes, (b) at most 1 defective bike (c) at least 2 defective bikes. [given $e^{-3} = 0.4979$].
- iii) Mr Patil plans to save for his daughter's marriage . He wants to accumulate a sum of Rs 4,00,000 at the end of 4 years. How much should he invest at the end of each year from now . If he can get interest compounded at 10% p. a. ? [given $(1.1)^4 = 1.4641$].

Q-6. (B) . Attempt any TWO of the following :

[8]

- i) Solve the following L.P.P.
Minimize $Z = 10x + 15y$, Subject to $2x + 3y \leq 12$, $2x + y \geq 6$, $x \geq 0$, $y \geq 0$
- ii) Taking population A as a standard population , find the standard death rate of the two districts . Which of the districts is healthier.

Age group	0 -10	10-55	Above 55
Population A	4000	6000	3000
Death in A	48	18	90
Population B	3000	7000	4000
Death in B	27	28	72

- iii) Six jobs go first over the machine M and then over machine N, one at a time. The time schedule for the task is given below. Determine the sequences of the jobs, which will minimize the processing time. Also find the total elapsed time and the idle time for the machines.

Machines	Jobs					
	A	B	C	D	E	F
M	5	9	4	7	8	6
N	7	4	8	3	9	5
